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The Mathematics and Marketing of Dead Chip Programmes: Finding and Keeping the Edge

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ABSTRACT *A general perception exists among casino executives that the premium players contribute a disproportionate share of profits. Consequently, to attract and retain high-end customers, many casinos are using 'dead chip' programs and other incentives. Dead chips are non-negotiable chips that are not exchangeable or redeemable and cannot be used for any purpose except to gamble. Very often the result of such a program is to reduce the effective statistical advantage of the house on games played. This article provides a mathematical framework to determine the effective house advantage under dead chip programs with applications to a variety of games and discusses the marketing and management implications of dead chip programs in light of these results.*

Introduction

Casino executives often perceive that premium or high-end players account for a disproportionately large share of revenues and profits. An article by Innis *et al.* (2003) quotes Bill Eadington—a respected gaming scholar—as saying that just ten high-rollers may account for as much as one-third of baccarat revenues for Las Vegas's major casinos. Binkley (2001) contends that premium players are essential for a casino to produce cash flows commensurate with the large capital outlays that go into building a mega-resort. This long-standing perception of high-rollers being saviors of a casino establishment has recently been questioned by quite a few writers (e.g. MacDonald, 2001a; Lucas *et al.*, 2002; Watson and Kale, 2003). Very often it is the lack of understanding of the mathematical underpinnings behind various high-roller programs that sustains the illusion of premium players as the casino industry's most prized segment. One such program deserving mathematical scrutiny is the dead chip program.

Dead chips are non-negotiable gaming chips that can be used for gambling but cannot be exchanged for regular chips or cash. In a typical program the player purchases a certain amount of non-negotiable chips at a discounted rate, usually through a voucher called a chip warrant, and uses these chips for wagers just like normal chips. Once the player has used all the non-negotiables, the remaining regular chips from winning wagers may be redeemed for cash. Some casinos, particularly in Asia and Australia, use dead chip programs as a marketing strategy to attract higher-end players. The general idea is to give players a bonus or refund on non-negotiable gaming chips in return for a commitment to a certain level of play. These programs can be effective, but because playing with dead chips alters the game advantage, a casino must be

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careful in setting appropriate bonus amounts. Awarding a bonus or refund can sometimes result in an effective house advantage that is too low to offset operating costs, and may even result in the player having the edge.

Unfortunately, little is known about the precise effect of dead chips on the statistical advantage for different games. For the most part, casinos have opted for ad hoc procedures in determining the amount of the bonus or rebate they are willing to award in a given situation. Further, casinos are reluctant to use non-negotiables for games other than baccarat, a traditional favourite among high-rollers and the game for which casinos have some comfort when it comes to the use of dead chip programs. Due to increasing competition for high-end players, these programs are receiving more attention and the need for understanding their impact on the casino's bottom line has never been greater.

This article provides a general mathematical framework for assessing the effective house advantage under dead chip programs for a variety of games and wagers. The results suggest that many casinos do not adequately factor in the house advantage when offering dead chip programs. Specifically, this article has three broad objectives: (1) to briefly review the extant literature on marketing to high-rollers with particular emphasis on relevant mathematics; (2) to explain the underlying math behind dead chip programs; and (3) to discuss the resulting marketing and management implications.

Literature Review

Over the last decade or so, a few authors have addressed casino dead chip programs, but mathematical understanding of such programs is limited. In discussing marketing and management issues on the risks of dealing to high-end players, Eadington and Kent-Lemon (1992) provide an approximate formula for the effective house advantage in baccarat when using a dead chip discount. MacDonald (1996) illustrates the potential impact of these programs on casino revenues using examples for baccarat and roulette; a similar discussion appears in Kilby and Fox (1998). Hannum and Cabot (2001) offer a precise formula, without supporting evidence, for computing the house advantage given a dead chip bonus for baccarat. In each of these presentations, the mathematical results are limited to simple wagers involving a single winning outcome and for a single round of dead chip purchases, with examples from only baccarat or roulette. There is no generally applicable framework that allows for calculation of the effective statistical advantage resulting from the use of a dead chip program with possibly multiple rounds of purchases under a variety of games and wager scenarios.

On a wider plane, there has been some recent discussion on the impact of high-rollers on a casino's bottom line from the mathematical angle. MacDonald (2001a) cites examples of several companies blaming the lower than expected table-game hold for their poor quarterly or half-yearly results. These poor results, he goes on to explain, are brought about by dealing high-limit baccarat to a select few (predominantly Asian) customers.

In a mathematical analysis of the profitability of premium players, Lucas *et al.*, (2002, p. 70) conclude: 'Gaming executives should also be aware that high-rollers can and do win, as premium play is often characterised by risk and volatility with a potentially small contribution margin. The slight profit margins combined with the volatility of the game can produce extreme results within a given financial quarter.' The authors demonstrate that a player betting $3,000 per hand of baccarat for nine hours contributes a mere $280 toward a casino's fixed costs. This is assuming the theoretical house advantage holds through the duration of play.

Watson and Kale (2003) use aggregate data from the Australian gaming industry to estimate the risk and profitability associated with casinos' major market segments. The authors conclude: 'At a time when many casinos are busy building or refurbishing multi-million dollar facilities to cater exclusively to "mobile customers" (high-rollers), this article espouses a fresh and less risky perspective. Casinos do not necessarily have to engage in one-upmanship to woo the fickle-minded high-rollers' (p. 100). In catering to premium players, Bowen and Makens (1999) argue: 'The expense for every player should be justified by the return it will bring.' Clearly, a more sound understanding based on mathematical probabilities is required for a casino to effectively tailor its marketing programs toward high-rollers.

Effective House Advantage

Dead Chip Programs

Dead chip programs are usually targeted toward 'junkets'. A junket is a group of players who travel to a casino specifically for the purpose of gaming. The travel is prearranged through a junket representative and the players' costs associated with travelling to and staying at the casino are absorbed by the casino (Kilby and Fox, 1998, p. 339). MacDonald (1996) describes the general process of a typical dead chip program. A junket group arrives at a casino and players deposit front monies with the casino in exchange for chip purchase vouchers, also known as cheque credits or chip warrants. The vouchers are then used to purchase non-negotiable chips, which the players use for wagers during the course of play. Losing wagers are taken by the dealer while winning wagers are paid in regular ('live') chips and the original wager remains as non-negotiable chips. Once a player has exhausted his (most junket players are men) non-negotiable chips, he takes his live chip winnings and exchanges these for more non-negotiable chip vouchers at the casino cage. The value of this exchange is recorded and added to the player's/group's non-negotiable chip purchase voucher schedule. When the junket group departs, the non-negotiable chip purchase voucher is totalled and all remaining non-negotiable chip purchase vouchers are deducted as well as any non-negotiable chips held. This provides a value for the turnover—the cumulative amount of money wagered—upon which a commission is paid to the junket group operators. All chip purchase vouchers and chips are converted to cash or deducted from personal cheques held with a new cheque value written out, which may or may not include commission. The commission payment, then, is calculated essentially on the loss of non-negotiable chips over the period.

A few of the larger casinos in jurisdictions outside the Asia-Pacific region have recently implemented or are considering implementing dead chip programs for premium players. Kilby and Fox (1998) describe a version that surfaced in Las Vegas in which players are given a 'bonus' of additional free non-negotiable chips at the time of purchase. A player wagering at baccarat, for example, may receive $103,000 in non-negotiable chips for his $100,000 in cash, a 3% dead chip bonus. Subsequent purchases may carry a reduced bonus. Thus, once the \$103,000 in non-negotiable chips is gone, the player may receive \$102,000 in non-negotiable chips for the second purchase of \$100,000 in cash (2% bonus). For the third and all subsequent purchases of \$100,000 in cash, the player may be given \$101,000 in non-negotiables (1% bonus). Note that the player receives the entire buy-in amount in dead chips, and not, for example, \$100,000 in negotiable chips and \$3,000 in dead chips. Although it is obvious the effect of such a bonus program is to reduce the advantage the casino holds over the player, the precise amount of the effect is not immediately evident. Thus far, only Kilby and Fox (1998) have provided some examples for baccarat bets.

The reduction in house advantage associated with the dead chip bonus model is a function of the average length of time a chip will be wagered before lost, which in turn depends on the probability of losing the wager. The logic can be seen clearly with a simple coin-tossing experiment. Consider a wager on the outcome heads in a series of tosses of an honest coin. With heads and tails equally likely, a bet on heads will lose every other toss on average; thus the average number of tosses before the wager is lost is two. If the coin is biased so that the probability of heads is equal to .90, then the average number of tosses before a wager on heads is lost is ten. Generally, if P_L is the probability of losing for each of a series of independent wagers, then $1/P_L$ is the average number of trials (wagers) until a bet is lost. This result can be viewed in terms of the well-known geometric probability model (Freund, 1973). If p represents the probability of success on each of a series of independent Bernoulli trials and X = the number of trials until the first success occurs, then the expected value of X is equal to $1/p$. In double-zero roulette, for example, a straight-up single number bet will last on average $1/(37/38) = 1.027$ trials and an even-money wager will last on average $1/(20/38) = 1.90$ trials. Thus, the ratio of actual turnover to non-negotiable losses is a function of the probabilities associated with the individual game and particular wager placed. For roulette this ratio is close to one on single number straight-up wagers, close to one and a half for dozens and columns bets, and close to two for even-money wagers.

This helps explain why in a game like baccarat, casino operators are willing to pay commissions—usually 1.5% to 2.0%—that are greater than the house advantage (which is around 1.2% to 1.8%) on non-negotiable chips. This is because for baccarat, the effective house advantage on non-negotiable chips can be viewed as being twice that of the normal value due to the ratio of actual versus non-negotiable turnover being about 2:1 (slightly more for banker bets; slightly less for player bets). However, there is no published literature that provides a precise mathematical basis for calculating the necessary dead chip bonus for a desired effective house advantage.

Basic Mathematics

We precede our presentation of the fundamental theorem with the reasoning for a specific example. Suppose a player pays \$100,000 on buy-in and receives \$101,500 worth of non-negotiable chips, reflecting a 1.5% bonus. Assume further this player wagers these non-negotiables, one chip per wager, only on the baccarat banker, with probabilities of winning and losing equal to $P_W = .458597$ and $P_L = .446247$ respectively (probability of tie is .095156). On average, it will take 227,452.7 wagers (101,500/.446246) before all the non-negotiable chips are lost, after which time the player will have received 99,093.7 live chips for wagers won (.458597 × 227,452.7 × .95). Thus the casino profit is \$100,000–\$99,093.7 = \$906.30, equating to an effective house advantage of 0.40% (906.3/227,452.7 = .0040). This reasoning generalises to the following result.

Fundamental theorem of dead chips. Suppose a player receives $X(1 + B)$ non-negotiable chips in exchange for X dollars, so that B represents the dead chip bonus. Suppose further the player wagers these non-negotiable chips until lost, one per wager on a series of independent wagers each with probability of winning w units equal to P_W and probability of losing one unit equal to P_L. It is assumed that all wagers are made with non-negotiable chips; losing wagers are taken by the dealer, and winning wagers are paid in regular chips with the original wager remaining as a non-negotiable chip. Then,

1. the effective house advantage is given by

$$HA^* = HA - \frac{B}{1+B}P_L.$$

2. If $B > \dfrac{HA}{wP_W}$ then $HA^* < 0$.

Proof. To prove (1), note that on average a non-negotiable chip unit will be wagered $1/P_L$ times before it is lost, and therefore the average number of units wagered with $X(1 + B)$ non-negotiable chips before all are lost is $X(1 + B)/P_L$. Thus the player's expected win is $[X(1 + B)/P_L]wP_w$ units and the house retains $X - [X(1 + B)/P_L]wP_W$ units. The effective house advantage, then, is

$$HA^* = \frac{X - [X(1+B)/P_L]wP_W}{X(1+B)/P_L} = \frac{P_L - wP_W - BwP_W}{1+B}.$$

Using the fact that $HA = P_L - wP_w$, this becomes

$$HA^* = \frac{HA - BwP_WB}{1+B} = \frac{HA - B(P_L - HA)}{1+B} = HA - \frac{B}{1+B}P_L.$$

This proves (1); the proof of (2) follows easily.

Note that (2) gives the maximum bonus possible without giving the player the advantage. Tables 1 and 2 illustrate this with examples for wagers in baccarat, craps, and roulette. Margin of error figures for selected numbers of trials are included in the tables to shed light on volatility and permit calculation of 95% confidence limits for the actual win percentage. Table 1 and Table 2 explain these results.

Multiple Rounds, Multiple Winning Outcomes, Mixtures of Wagers

The fundamental theorem can be extended to results for multiple rounds of dead chip purchases, wagers involving multiple winning outcomes (for example, slots, keno, video poker, and certain bets in craps), and mixtures of wagers. These extended results are given below. Proofs are analogous to that of the preceding fundamental theorem and are omitted. In the results that follow, it is assumed that all wagers are made with non-negotiable chips, losing wagers are

Table 1 Effective house advantages*—baccarat and craps

	Baccarat		Craps	
	Banker[1]	Player[1]	Pass line	Don't pass[1]
House advantage	1.17%	1.36%	1.41%	1.40%
Margin of Error[2]				
n = 10,000	± 1.95%	± 2.00%	± 2.00%	± 2.00%
n = 100,000	± 0.62%	± 0.63%	± 0.63%	± 0.63%
n = 1,000,000	± 0.19%	± 0.20%	± 0.20%	± 0.20%
Dead chip bonus	HA*	HA*	HA*	HA*
0.00%	1.17%	1.36%	1.41%	1.40%
0.25%	1.05%	1.24%	1.29%	1.28%
0.50%	0.92%	1.11%	1.16%	1.15%
0.75%	0.80%	0.99%	1.04%	1.03%
1.00%	0.68%	0.86%	0.91%	0.90%
1.25%	0.56%	0.74%	0.79%	0.78%
1.50%	0.44%	0.62%	0.66%	0.65%
1.75%	0.32%	0.49%	0.54%	0.53%
2.00%	0.20%	0.37%	0.42%	0.41%
2.25%	0.08%	0.25%	0.30%	0.29%
2.50%	− 0.03%	0.13%	0.18%	0.17%
2.75%	− 0.15%	0.01%	0.06%	0.05%
3.00%	− 0.27%	− 0.11%	− 0.06%	− 0.07%
Max bonus	2.43%	2.77%	2.87%	2.85%

1 Figures given excluding ties.
2 The maximum difference (two standard deviations) between the actual and theoretical win percentage, with 95% confidence, after the given number of trials. For example, there is a 95% probability that after 100,000 (equal size) baccarat banker bets the actual casino win percentage will be within ± 0.62% of the house advantage. For normal baccarat play, this means there is 95% confidence that the actual win rate for 100,000 banker bets will be between 0.55% and 1.79%.
* Effective house advantage is given by HA* . For example, a 1.75% dead chip bonus given to a player betting on the baccarat banker will effectively lower the house advantage from the normal 1.17% (excluding ties) to 0.32%. The maximum bonus percentage that can be given before the player has the advantage on this bet is 2.43%.

taken by the dealer, and winning wagers are paid in regular chips with the original wager remaining as a non-negotiable chip.

Multiple rounds. Suppose a player purchases k rounds of non-negotiable chips, exchanging X_i dollars for $X_i(1 + B_i)$ non-negotiable chips on round i, for $i = 1, 2, \ldots k$. Thus $B_1, B_2, \ldots B_k$ represent the dead chip bonuses on each of these purchases. Suppose further the player wagers these non-negotiable chips until lost, one per wager on a series of independent wagers each with probability of winning w units equal to P_W and probability of losing one unit equal to P_L. Then the effective house advantage after k rounds is given by

$$HA^* = HA - \frac{\sum_{i=1}^{k} X_i B_i}{\sum_{i=1}^{k} X_i(1 + B_i)} P_L.$$

Table 2. Effective house advantages* —roulette

	Single-zero roulette			Double-zero roulette		
	Evens[1,2]	Dozens	Single #	Evens[1]	Dozens	Single #
House advantage	2.70%	2.70%	2.70%	5.26%	5.26%	5.26%
Margin of error[3]						
n = 10,000	± 2.00%	± 2.81%	± 11.68%	± 2.00%	± 2.79%	± 11.53%
n = 100,000	± 0.63%	± 0.89%	± 3.69%	± 0.63%	± 0.88%	± 3.64%
n = 1,000,000	± 0.20%	± 0.28%	± 1.17%	± 0.20%	± 0.28%	± 1.15%
Dead chip bonus	HA*	HA*	HA*	HA*	HA*	HA*
0.00%	2.70%	2.70%	2.70%	5.26%	5.26%	5.26%
0.25%	2.57%	2.53%	2.46%	5.13%	5.09%	5.02%
0.50%	2.45%	2.37%	2.22%	5.00%	4.92%	4.78%
0.75%	2.32%	2.20%	1.98%	4.87%	4.75%	4.54%
1.00%	2.19%	2.03%	1.74%	4.74%	4.59%	4.30%
1.25%	2.07%	1.87%	1.50%	4.61%	4.42%	4.06%
1.50%	1.94%	1.70%	1.26%	4.49%	4.25%	3.82%
1.75%	1.82%	1.54%	1.03%	4.36%	4.09%	3.59%
2.00%	1.70%	1.38%	0.79%	4.23%	3.92%	3.35%
2.25%	1.57%	1.22%	0.56%	4.11%	3.76%	3.12%
2.50%	1.45%	1.05%	0.33%	3.98%	3.59%	2.89%
2.75%	1.33%	0.89%	0.10%	3.85%	3.43%	2.66%
3.00%	1.21%	0.73%	–0.13%	3.73%	3.27%	2.43%
Max bonus	5.56%	4.17%	2.86%	11.11%	8.33%	5.71%

1 Any of the even-money wagers (covering 18 numbers).
2 No *en prison*.
3 The maximum difference (two standard deviations) between actual and theoretical win percentage, with 95% confidence, after the given number of trials. For example, there is a 95% probability that after 1,000,000 (equal size) even-money wagers in single-zero roulette, the actual casino win percentage will be within ± 0.20% of the house advantage. For normal roulette play, this means there is 95% confidence that the actual win rate for 1,000,000 even-money wagers on the single-zero wheel will be between 2.5% and 2.9%.
* Effective house advantage is given by HA* . For example, a 2.50% dead chip bonus given to a player betting on even-money wagers in single-zero roulette will effectively lower the house advantage from the normal 2.70% to 1.45%. The maximum bonus percentage that can be given before the player has the advantage on this bet is 5.56%.

Note that if the purchase amounts (X_i) on each round are equal, then the above result reduces to

$$HA^* = HA - \frac{\bar{B}}{1+\bar{B}} P_L,$$

where $\bar{B} = (\Sigma B_i)/k$.

Multiple winning outcomes. Assume the same framework as in Theorem 1, except that each wager has $(m+1)$ possible outcomes (payoffs) $w_1, w_2, \ldots, w_m$, and -1, with probabilities $P_1, P_2, \ldots P_m$, and P_L, respectively, with $w_i \geq 0$ for $i = 1, 2, \ldots m$, and normal house advantage given by $HA = P_L - \sum_{i=1}^{m} w_i P_i$. Then,

1. the effective house advantage is given by

Table 3. Multiple winning outcomes*

Wager		Bonus = 1.5%		Bonus = 4.0%		Bonus = 8.7%[1]	
Payout	Prob	Wagers[2]	Win[2]	Wagers[2]	Win[2]	Wagers[2]	Win[2]
− 1	0.750	101,500	0	104,000	0	108,696	0
0	0.100	13,533	0	13,867	0	14,493	0
1	0.100	13,533	13,533	13,867	13,867	14,493	14,493
2	0.045	6,090	12,180	6,240	12,480	6,522	13,043
100	0.005	677	67,667	693	69,333	725	72,464
HA = 6.00%		135,333	93,380	138,667	95,680	144,928	100,000
		House Win	6,620	House Win	4,320	House Win	0
		HA* = 4.89%		HA* = 3.12%		HA* = 0.00%	

1 Maximum bonus without giving player the edge is 8.695652174% (used in calculations).
2 Player wagers made and live chips won on average assuming 100,000 unit buy-in.
* Effective house advantage is given by HA* . For example, a 4.0% dead chip bonus given to a player betting on this multiple outcome wager (two left columns) will effectively lower the house advantage from the normal 6.00% to 3.12%. Assuming a 100,000 unit buy-in, this player will make on average 138,667 wagers with the 104,000 dead chips received, winning 95,680 negotiable chips, resulting in a casino win of 4,320 units: 4,320/138,667 = .0312.

$$HA^* = HA - \frac{B}{1+B}P_L.$$

2. If $B > \frac{HA}{P_L - HA}$, then $HA^* < 0$.

Table 3 provides an example of the multiple winning outcomes result using $m = 4$ and shows the details for three dead bonus percentages: 1.5%, 4.0%, and 8.7%. In this example, with a relatively large probability of losing and normal statistical advantage equal to 6.0%, the maximum bonus without giving the player the advantage is 8.70% [from result (2) above].

Thus far, we have assumed dead chips are used on a single type of wager (or independent wagers with the same payoff probability distribution). The next result shows how to compute the effective statistical advantage when dead chips are used for a mixture of different bets.

Mixtures of wagers. Suppose a player receives $X(1 + B)$ non-negotiable chips in exchange for X dollars, so that B represents the dead chip bonus. Suppose further the player wagers a fraction f_i of these non-negotiable chips on wager i, for $i = 1, 2, \ldots s$, one chip per independent wager, until lost. Let P_{Wi} and P_{Li} represent the probabilities of winning and losing, and w_i the winning payoff associated with wager i. Then the effective house advantage is given by

$$HA^* = \frac{1-(1+B)\sum f_i(\frac{w_i P_{W_i}}{P_{L_i}})}{(1+B)\sum(\frac{f_i}{P_{L_i}})} = \sum_{i=1}^{s} r_i.HA_i^*,$$

where $r_i = \frac{f_i/P_{L_i}}{\sum_{j=1}^{s} f_j/P_{L_j}}$ and HA^*_i is the effective house advantage for wager i.

Table 4. Mixtures of bets*

Roulette (single-zero) mix—1.5% bonus

	P_L	P_W	w	HA	f_i	r_i	HA_i*
Evens[1]	0.5135	0.4865	1	2.70%	0.200	0.294	1.94%
Dozens	0.6757	0.3243	2	2.70%	0.200	0.223	1.70%
Corner	0.8919	0.1081	8	2.70%	0.200	0.169	1.38%
Split	0.9459	0.0541	17	2.70%	0.200	0.159	1.30%
Single #	0.9730	0.0270	35	2.70%	0.200	0.155	1.26%
							HA* = 1.59%

Baccarat[2] – 50% banker, 50% player—1.5% bonus

	P_L	P_W	w	HA	f_i	r_i	HA_i*
Banker	0.4932	0.5068	0.95	1.17%	0.500	0.507	0.44%
Player	0.5068	0.4932	1	1.36%	0.500	0.493	0.62%
							HA* = 0.53%

Baccarat[2] – 75% banker, 25% player—1.5% bonus

	P_L	P_W	w	HA	f_i	r_i	HA_i*
Banker	0.4932	0.5068	0.95	1.17%	0.750	0.755	0.44%
Player	0.5068	0.4932	1	1.36%	0.250	0.245	0.62%
							HA* = 0.48%

Baccarat[2] and roulette (single-zero) mix—1.5% bonus

	P_L	P_W	w	HA	f_i	r_i	HA_i*
Banker	0.4932	0.5068	0.95	1.17%	0.250	0.291	0.44%
Player	0.5068	0.4932	1	1.36%	0.250	0.283	0.62%
Evens[1]	0.5135	0.4865	1	2.70%	0.250	0.279	1.94%
Single #	0.9730	0.0270	35	2.70%	0.250	0.147	1.26%
							HA* = 1.03%

1 Any of the even-money wagers (covering 18 numbers); no *en prison*.
2 Figures given excluding ties.
* Effective house advantage is given by HA* . For example, given a dead chip bonus of 1.5%, a player betting equal proportions of dead chips on baccarat banker, baccarat player, roulette even-money and roulette single number wagers will result in an effective house advantage of 2.10%.

Table 4 illustrates the calculation of effective statistical advantage for several mixtures of roulette and baccarat wagers.

Dead Chip Commission after Play

In some programs, the dead chip bonus is awarded as a commission in cash after the termination of play rather than at the time of the buy-in. In this situation, the effective house advantage computation is straightforward, and can be shown to be $HA^* = HA - BP_L$.

Note that giving the dead chip bonus as a commission at the end of play will always result in a lower effective house advantage than giving the bonus at the time of buy-in, all else equal. The difference between the effective house advantage under the commission model and the original model is easily derived, and is given by:

$$\Delta = \frac{B^2}{1+B} P_L.$$

Marketing and Management Implications

Casino management and marketing strategies should be based on mathematically sound business decisions with a mechanism to assess the long-term theoretical win or loss to the business (Bowen and Makens, 1999; Eadington and Kent-Lemon, 1992; MacDonald, 1999). The results in Tables 1 and 2 provide casino management with reference values for appropriate dead bonus percentages and upper limits for these bonuses that could be made available to players of baccarat, craps and roulette. As with all expected value calculations, these figures apply in the long run and as such can serve as the basis for sound business decisions. It would be unrealistic, however, to presume that short-term results will conform precisely to the theoretical advantage. As MacDonald (2001a) rightly points out,

> Baccarat is a game of chance and so fortunes can, and do, pass both ways across the gaming tables. The house advantage on baccarat averages a very low 1.3% to make this close to a true even money bet every time cards are drawn for a new hand. An interesting statistic is that 14,981,640 hands need to be played for there to be a 95% confidence interval of results falling between a win rate of 1.25% and 1.35%. If you multiply this number by a bet of $100,000 you require $1,498,164,503,243 in turnover for win rate "certainty." An astounding number in anyone's view!

Although the laws of probability warrant that the actual win percentage will converge on the theoretical win percentage, management needs to be concerned about potentially large deviations from the expectation that may occur in the short run. The margin of error figures in Tables 1 and 2 provide upper bounds, with 95% confidence, on the size of this deviation for several numbers of trials (hands) and can be used to obtain 95% confidence limits for the actual win rate. About baccarat volatility, MacDonald (2001b) concludes: 'The level of turnover required for the actual win percentage to fall within a small level of tolerance from the mean can, however, be astronomical, so analysts and senior management need to have very strong stomachs if involved in baccarat high end play.' Even if the house advantage miraculously holds through the duration of play, the casino will still have to foot the bill for room, food and beverage, airfare allowance and gaming taxes.

Thanks to the higher house advantage in roulette, a casino is on more solid ground when it offers dead chip bonus to roulette players. In fact the maximum bonus in roulette for a player betting 'evens' (any of the even-money bets, such as red or black) could be twice as high as a baccarat player betting on the banker. Having thus determined the house advantage and the resulting bonus a casino could safely live with for the various games, bonuses should be based on specific games as well as a punter's total buy-in for dead chips. Of course, the house advantage can be even higher in other games—keno and some slots, for example—but these are not usually the games of choice for high-rollers.

The situation gets most uncertain from a casino's standpoint when dead chip players are allowed to play a variety of games. Since house advantage is a statistical function of the particular game played and particular wagers made, establishing a realistic bonus for a player betting on both blackjack and craps becomes an extremely unwieldy proposition. Unless the precise mixture of games to be played can be controlled in some way, a casino may need to be more conservative if allowing a mixture, perhaps using as the basis for the dead chip bonus the game that would result in the lowest bonus percentage. Otherwise, it makes sense to design dead chip bonus programs under the condition that the chips could be used for play in only one specified game. Note too, that for games that involve an element of skill, the house advantage will vary depending on the player. In such situations, calculations assuming optimal strategy will provide a baseline reference for skilled players, but the actual house advantage will be higher for those players who do not use optimal strategy. Of course, for many games, including baccarat, the game most associated with dead chip program play, skill is not an issue.

Another implication of the calculations presented herein relates to the wisdom of providing the dead chip bonus at the time of buy-in as opposed to at the termination of play. Giving a dead chip bonus at the end of play will always result in a lower effective house advantage than giving the same bonus at the time of buy-in. Given slim operating margins, casinos are better off giving commission at the time of buy-in. Although many players may be unaware of the mathematical intricacies surrounding this choice, more savvy players and junket representatives will seek out dead chip programs offering commission at the end of play rather than at buy-in.

Besides considering the win rate volatility in deciding commissions, other factors such as volume, bet-limits, incentives and credit risk also need to be factored in. In setting betting limits associated with high-rollers, one should pay attention to capital reserves of the company, the company's propensity for risk, fixed and variable expenses associated with the operation, the total handle, and so on. Not too many companies (or their shareholders) would be thrilled in the event of being forced to utilise their capital reserves to pay for table losses or fixed expenses. Incentives accrued to high-rollers should be accurately estimated in order to arrive at the bonus figure to award on dead chips. As MacDonald (2001a) observes, incentives typically total somewhere between 50% and 70% of theoretical win. This allows little room for bonuses or commissions. Bear in mind that dealing with high net-worth individuals means that you are often negotiating with very influential businesspeople who demand (and usually get) the best of everything. Costs associated with airfare, hotel suites, golf course fees, and food and beverages do add up, and offering these as comps cuts significantly into the casino's theoretical win. Furthermore, the credit risk associated with high-rollers needs to be factored in when determining the bonus amount. As the industry saying goes, 'You have to win the money twice.' Since most high-rollers play on credit, casinos have to win the money over the tables *and* they also have to win by getting paid, which may be neither very prompt nor certain.

The results presented in this article shed further light on the profitability and risk associated with high-end play. While the focus is on dead chip programs, the findings here corroborate the views of other scholars who have studied the premium segment (Lucas *et al.*, 2002; MacDonald, 2001a; Watson and Kale, 2003). In offering incentives to high-rollers, most casinos sacrifice a substantial portion of their house advantage, thus creating a significant dent in profitability. Watson and Kale (2003) have suggested that instead of continuing to offer high incentives primarily to premium players, casinos should consider marketing strategies and programs that target the $25 to $200 bettors. These mid-level punters constitute the backbone of the casino gaming business; offering rebate and discount programs will lower the price of the game for these players and, at the same time, have a positive impact on casino revenues and customer retention.

Although it is natural to view the model presented here in terms of a player's single visit to the casino, it would not be difficult to extend the analysis to apply to multiple visits. With the current emphasis on customer relationship management (CRM), it is advisable to consider player profitability over the entire length of a player's relationship with the casino. Given the volatility associated with short-term play of the high-end segment, it would make sense to make every attempt toward enhancing the lifetime value of high-rollers. At a time when casinos all over the world fiercely compete for a share of the high-rollers' wallets, engendering loyalty among high-rollers is a formidable task.

A logical next step for future research would be to evaluate the mathematics presented in this article using actual expenditure data and perhaps identifying and incorporating other important, unrecognised influences that shape gambling patterns. A player's betting strategy is influenced by his personality (e.g. sensation-seeking and risk-taking) as well as situational variables (past wins and losses, behaviours of other players, and so on). These will impact the size and sequence of betting, and will, therefore, impact the gambling provider's bottom line. Whether pursuing the high-end or the low-end of the consumer spectrum, a solid understanding of consumer behaviour as well as mathematics is needed to design effective promotions.

Conclusion

Dead chip programs have been used by casinos for some time—particularly in the Asia-Pacific region—but the mathematics underlying these programs has not been well-understood. Comprehending the effect these programs have on game advantage is crucial to the casino's ability to offer an attractive dead chip package while still conducting sound business. As the popularity of dead chip programs grows, so does the need for a clear understanding of the precise impact of this marketing initiative on the mathematical edge. Recent history of several casinos ending up on the losing side of the table vis-à-vis high-rollers underscores the importance of thoroughly understanding the mathematical realities behind incentive programs. Casinos will do well to heed the cautionary advice of Lucas *et al.* (2002, p. 75) who write: 'Many casino executives assume that the high-roller segment is inherently profitable. Combine that assumption with a misunderstanding of gaming mathematics and you have the makings of a downward profit spiral.'

This article provides an explicit formula for computing the effective house advantage for a given dead chip bonus percentage and has demonstrated that the maximum bonus figure before giving the player the advantage is a function of the game on which the chips will be wagered. All else being equal, the effective house advantage is greater when commission is given at the time of buy-in as opposed to at the conclusion of play. Since the determination of an appropriate bonus becomes a challenging task when dead chips are wagered on several different games, it may make sense for casinos to restrict dead chip program players to a particular game.

One needs to bear in mind that mathematics only indicates that the actual win rate will be close to the house advantage after a very large number of trials. In the short term, deviations from the theoretical win will inevitably occur, making dealing to high-rollers an inherently risky proposition for most casinos. Decisions on commission should therefore be made in conjunction with decisions on bet limits, other incentives, and credit risk. All these decisions should be formulated against the backdrop of a casino's financial reserves, its tolerance for risk, shareholder expectations, and expected turnover volumes. To quote MacDonald (2001a) again, 'This (high-roller segment) can be the sexy, sharp end of the business but at the end of the day it is low margin and high risk. Lots of risk!'

References

Binkley, C. 2001. 'Reversal of fortune', *Wall Street Journal*, September 7, pp. A1, A8.

Bowen, J.T. and Makens, J.C. 1999. 'Promoting to the premium player: An evolutionary process', in Eadington, W.R. and Cornelius, J.A. (eds) *The Business of Gaming: Economic and Management Issues*. Institute for the Study of Gambling and Commercial Gaming, University of Nevada, Reno: pp. 229–39.

Eadington, W.R. and Kent-Lemon, N. 1992. 'Dealing to the premium player: Casino marketing and management strategies to cope with high risk situations', in Eadington, W.R. and Cornelius, J.A. (eds) *Gambling and Commercial Gaming: Essays in Business, Economics, Philosophy and Science*. Institute for the Study of Gambling and Commercial Gaming, University of Nevada, Reno: pp. 65–80.

Freund, J.E. 1973. *Introduction to Probability*. Dover Publications, New York.

Hannum, R.C. and Cabot, A.N. 2001. *Practical Casino Math*. Institute for the Study of Gambling and Commercial Gaming, University of Nevada, Reno.

Innis, M., Healy, T. and Keeley, B. 2003. 'Hunting for whales', *Far Eastern Economic Review*, December 4, pp. 57–58.

Kilby, J. and Fox, J. 1998. *Casino Operations Management*. Wiley, New York.

Lucas, A.F., Kilby, J. and Santos, J. 2002. 'Assessing the profitability of premium players', *Cornell Hotel and Restaurant Administration Quarterly*, August, pp. 65–78.

MacDonald, A. 1996. 'Non negotiable chips: Practical procedures and relative costs of non negotiable play on table games', *Casinos 2000*, Article 11 [www.urbino.net].

MacDonald, A. 1999. 'Player loss: How to deal with actual loss in the casino industry', in Eadington, W.R. and Cornelius, J.A. (eds) *The Business of Gaming: Economic and Management Issues*. Institute for the Study of Gambling and Commercial Gaming, University of Nevada, Reno: pp. 217–27.

MacDonald, A. 2001a. 'Dealing with high-rollers: A roller coaster ride?', Article posted 18 March 2001 [www.urbino.net].

MacDonald, A. 2001b. 'Game volatility at Baccarat', *Gaming Research and Review Journal*, 6(1), pp. 73–7.

Watson, L. and Kale, S.H. 2003. 'Know when to hold them: Applying the customer lifetime value concept to casino table gaming', *International Gambling Studies*, 3(1), pp. 89–101.